REPORT TITLE: Torsion Test

STUDENT: Ryan Fuld

GROUP: C

COURSE NO., SECTION NO.: ME2900-003

DATE PERFORMED: 2/28/07

DATE DUE: 3/14/07

DATE SUBMITTED: 3/14/07

OTHER GROUP MEMBERS THAT WORKED ON THIS EXPT.

1. Collin Hale
2. Andrij Harasewych
3. Brandon Lee
4. Adam Lupisella

FOR FACULTY USE

DATE RECEIVED:
QUIZ GRADE:
REPORT GRADE:
OVERALL GRADE:

COMMENTS:
Torsion Test

I. Objective:

To apply torsion tests in order to observe properties of certain steel specimens in pure shear; a rod that has been annealed, and a rod that has been cold worked.

II. Theory:

Torsion tests involve the application of a torque to a rod made up of a certain metal. When torque is applied, internal forces are created, causing the rod to twist. If the rod were cut into small disks, they would all move somewhat with respect to each other. That is, the disk behind the disk in front of it will flex with respect to the front one. As the distance from the applied torque increases, the disks will move a lesser amount, until the end of the rod, where there is, theoretically, no movement. This is true for the initial part of a stress strain curve, wherein the relationship between the two is linear. The goal of the torsion test is to measure the amount in which these theoretical disks move, allowing an analysis of the rod to take place. Within this analysis, a value for the modulus of rigidity can be calculated, which describes the strength of the metal, and more specifically, the response of the metal to a sort of shearing strain. In addition to this, the yield point of the specimen can be determined, which illustrates the point at which the specimen leaves the elastic range, and enters the plastic range, and acquires a permanent set, or deformation. In applying a torsion test, dimensions of the specimen come into play, in addition to the properties of the metal. These dimensions include the diameter, the length, and the polar moment of inertia of the cylindrical rod. Moreover, a torsion test will help analyze a certain structure in pure shear, rather than a general material.
Torsion can most easily be applied to and explained by the concept of a motor driving an appliance, such as an automobile. The motor will be attached to a shaft, which will, in turn, be geared to another shaft, with multiple bearings to hold the shafts in place. Furthermore, due to the load created by the motor, stress is developed, which tries to shear the driven shaft, as the gear teeth generate opposing loads; the tangential separating load is the greatest. The loads resist the torque generated by the motor, which creates shear stress that attempts to slice through the shaft; this is at a maximum nearest the bearings, perpendicular to the torque applied. Thus, the end result will be a twisting of the shaft. Specifically, in this experiment, a Model 1125 Instron Test Machine is used in order to twist the provided specimens. This is controlled by a computer, which collects data on the twisting angle and the torque induced in the specimen. With this data, a plot is created, of Torque vs. Time, creating a graph with a slope that can be used in order to find the modulus of rigidity, which will be explained in the derivation of the respective equation. The slope can be found by viewing specific data points collected by the computer in text files. This also allows calculations of stress and strain, using their respective equations.

It can be helpful to look further into the concept of torsion, considering additional factors in breakage. It is actually true that there are not only shear forces, but, in addition, simultaneous forces of tension acting upon the axis, which act on the specimen. This can become important when analyzing identical metals which have been formed differently, resulting in different atomic structures, as tension can cause differently formed metals to break differently, say, on slightly different angles. If it were to break perfectly straight, that could suggest higher tension forces. Lastly, the rods can break in different points, say, in the middle, or near the end. This is mostly dependant on the way the metal was formed, and if
the mixture of the metal is homogenous throughout the rod. If it is not, then it may likely break nearer to the end, rather than the middle, where it theoretically should break.

An explanation of the difference in failure of annealed and cold worked steels can be very useful in the analysis of the specimens. First off, failure occurs due to the fibers, or grains, moving and contacting each other. In the metal during the annealing process, the metal is heated and slow cooled, resultantly removing internal stresses, making it softer, and changing the material properties. The opposite is true for cold working, specifically cold rolling, wherein internal stresses are increased, causing it to be harder and more brittle, decreasing the ability to be stretched or twisted, but at the same time, making it stronger. This, in turn, would mean that a cold worked rod should fail prior to an annealed rod, but should have a yield point occurring at a higher internal torque.

In order to accurately determine values for the modulus of rigidity, shear stresses, and shear strains, equations must be derived which describe these values in terms of values which can be measured. However, the concept of shear stresses and shear strains must be clearly defined and understood in order to derive the respective formulas. Shear stress is formally explained by examining a small cross section of a member, where shear acts parallel or tangential to the face of the member, and thus perpendicular to the axis of the member. These are created by forces acting upon a differential area, as previously mentioned. Shear strain, however, is exactly the concept described wherein cross-sectional disks move in respect to each other. The accumulation of the movement of all the cross-sectional disks is equivalent to the deformation, or twisting of the rod. If a line is drawn on the rod, the angle between the original position of the line and the new position, which is the movement of the disks, is defined as the shear strain. Understanding these concepts eases the derivation of all necessary equations, including the modulus of rigidity. The derivation is as follows:
01. \[ \int \rho \, dF = T \]

Torque (T) is equivalent to the sum of the shearing moments, caused by shearing forces (dF) about a perpendicular distance (\(\rho\)). For a cylindrical specimen, the resultant torque can be expressed as an integral, wherein \(\rho\) is any distance within the radius (r). As shown in the figure, a reaction torque (T') exists to keep the cylinder in static equilibrium, due to the unseen torque (T), generated by the differential shear forces (dF) shown.

02. \[ \tau = \frac{dF}{dA} \]

\[ \rightarrow dF = \tau \cdot dA \]

The definition of shearing stress states that the shearing stress (\(\tau\)) is equivalent to any element of applied force (dF) subjected over an element of area (dA). The equation can be manipulated so that an expression for the differential force (dF) can be obtained.

03. \[ \int A \rho \tau \, dA = T \]

By rearranging the equation expressed in [02.], it can be seen that the differential force (dF) is equal to the product of the shearing stress (\(\tau\)) and the differential area (dA). By substituting this equation in for the differential force expressed in previously defined torque in [01.], it can be seen that the torque (T) can also be expressed as the area integral of the product of the perpendicular distance (\(\rho\)) and the shearing stress (\(\tau\)).

04. \[ \phi = \frac{c}{\rho} \]

\[ \rightarrow c = \phi \rho \]

By observing the geometry of cylindrical specimen after it has been subjected to torsion and applying trigonometric laws, it can be observed that, for small enough degrees of twist, the quotient of the arc length (c) and the perpendicular distance (\(\rho\)) is essentially equal to the tangent of the angle of twist, or deformation (\(\phi\)). Furthermore, for small enough angles, the tangent of any angle is very close to the actual value of the angle itself. This said, because the angle of deformation due to torsion is generally very minimal, the previously mentioned equation can be redefined into the form shown, wherein the angle of deformation (\(\phi\)) is equal to the arc length (c) divided by the perpendicular distance (\(\rho\)). A manipulated version of the equation is also shown, which provides an expression for the arc length (c). In the diagram, \(\rho'\) represents the distance from the center of the rod to the new position, \(q'\), of the point \(q\), after torsion has occurred.

05. \[ \gamma = \frac{c}{L} \]

Revisiting the geometry of a cylindrical specimen that has subjected to torsion allows an additional trigonometric observation to be made, wherein the arc length (c) divided by the entire length of the specimen (L) is equal to the shearing strain (\(\gamma\)). The more numerically correct relationship is actually the same as expressed in the description in [04.], however, the shear strain angle is also very minimal, allowing the same generalization to be made. The relationship can be seen in the figure below.
If the manipulated version of [04.] is substituted into the equation shown in [05.], an additional equation for shear strain ($\gamma$) can be expressed, wherein it is then equal to the product of the angle of twist ($\phi$) and a perpendicular distance from the center ($\rho$), divided by the entire length of the specimen ($L$). This said, the shear strain ($\gamma$) is then defined as a function of the perpendicular distance from the center of the specimen. This can further be visualized in the figure below, where the arc length ($c$) is the distance between $q$ and $q'$.

$$c = \phi \cdot \rho, \quad \gamma = \frac{c}{L} \implies \gamma = \frac{\phi \cdot \rho}{L}$$

As the perpendicular distance ($\rho$) approaches the value for the total radius ($r$), the torsional shearing stress ($\rho$) increases. This is because the shear strain ($\gamma$) was defined as the product of the angle of twist ($\phi$) and any perpendicular distance ($\rho$) within the total radius ($r$), all divided by the entire length ($L$). Furthermore, it can be observed that the maximum shear strain ($\gamma_{\text{max}}$) must then be then equal to the angle of twist ($\phi$) multiplied by the entire radius ($r$), divided by the length of the specimen ($L$). Moreover, this occurs when the perpendicular distance $\rho$ is equal to the radius of the cylinder ($r$). As shown in the figure in [06.], the shear strain $\gamma$ is maximum, as the distance between $q$ and $q'$ is actually the entirety of the arc length, which is the angle of deformation ($\phi$) multiplied by the radius ($r$). The relationship between the shear strain distribution ($\gamma$) and the changing radius, from any radius $\rho$ to the total radius $r$ can further be seen in the figure below.

The equation can be manipulated so that the rate of twist ($\theta$), which is defined as the angle of twist ($\phi$) per the length ($L$), is equal to the maximum shear stress ($\gamma_{\text{max}}$) divided by the radius of the specimen ($r$).

$$\phi \frac{L}{r} = \theta = \frac{\gamma_{\text{max}}}{r}$$

By substituting the manipulated version of [07.] into [06.], it is seen that the shear strain ($\gamma$) is equal to the ratio of any perpendicular distance from the center of the specimen ($\rho$) to the total radius of the specimen ($r$), multiplied by the maximum shear strain ($\gamma_{\text{max}}$).

$$\frac{\phi}{L} = \frac{\gamma_{\text{max}}}{r}, \quad \gamma = \frac{\phi \cdot \rho}{L} \implies \gamma = \frac{\gamma_{\text{max}} \cdot \rho}{r}$$
09. $\tau = G \cdot \gamma$

Hooke's Law states that the torsional shear stress ($\tau$) is equal to the modulus of rigidity ($G$) multiplied by the torsional shear strain ($\gamma$).

10. $\tau = \frac{\rho}{r} \cdot G \cdot \gamma_{\text{max}}$

By simply substituting the equation given in [08.] into the equation given by Hooke's Law shown in [09.], the torsional shear stress ($\tau$) can be expressed alternatively as the shown function of the perpendicular distance ($\rho$), the total radius ($r$), the modulus of rigidity ($G$), and the maximum shear strain ($\gamma_{\text{max}}$).

$$\gamma = \frac{\rho}{r} \cdot \gamma_{\text{max}}, \quad \tau = G \cdot \gamma \quad \rightarrow \quad \tau = G \cdot \frac{\rho}{r} \cdot \gamma_{\text{max}}$$

11. $\tau_{\text{max}} = G \cdot \gamma_{\text{max}}$

If Hooke's Law is revisited, an equation can be expressed which relates maxima. That is, since the shear stress ($\tau$) is defined as the product of the modulus of rigidity ($G$) and the shear strain ($\gamma$), the maximum shear stress ($\tau_{\text{max}}$) must then be equivalent to the product of the modulus of rigidity ($G$) and the maximum shear strain ($\gamma_{\text{max}}$).

12. $\tau = \frac{\rho}{r} \cdot \tau_{\text{max}}$

If a basic substitution is made, wherein the left side of the equation in [11.] is placed in equation [10.] for the instance where the right side of equation [11.] appears, an expression for the shear stress ($\tau$) is given, in terms of a chosen perpendicular distance ($\rho$), the total radius ($r$), and the maximum shear stress ($\tau_{\text{max}}$). The relationship here is the same as was seen with shear strain ($\gamma$) in [08.]

$$\tau_{\text{max}} = G \cdot \gamma_{\text{max}}, \quad \tau = \frac{\rho}{r} \cdot G \cdot \gamma_{\text{max}} \quad \rightarrow \quad \tau = \frac{\rho}{r} \cdot \tau_{\text{max}}$$

13. $\int \frac{\rho^2}{r} \cdot \tau_{\text{max}} \, dA = T$

By making use of the equation from [12.], and substituting it in the equation from [03.], an equation can be constructed which expresses the torque ($T$) as the area integral of the square of the perpendicular distance from the center of the cylindrical specimen ($\rho$) divided by the total radius ($r$) multiplied by the maximum torsional shearing stress ($\tau_{\text{max}}$). The equation can be manipulated in order to solve for the maximum torsional shearing stress ($\tau_{\text{max}}$). Since $\tau_{\text{max}}$ and the radius ($r$) are fixed constant values for any cylindrical specimen, they can simply be pulled outside of the integral.

$$\tau = \frac{\rho}{r} \cdot \tau_{\text{max}}, \quad \int \rho \cdot \tau_{\text{max}} \, dA = T \quad \rightarrow \quad \int \rho \cdot \frac{\rho}{r} \cdot \tau_{\text{max}} \, dA = T$$

14. $\int \rho^2 \, dA = J$

The area integral of the square of the perpendicular distance of from the center of a cylindrical specimen; some inner radius ($\rho$), is defined as the polar moment of inertia ($J$). Moreover, the polar moment of inertia is the distribution of the area about the centroid of the object, which can be calculated for various objects using certain equations.
If the equation in [14.] is plugged into the equation from [13.], an expression can be formulated which gives the maximum torsional shearing stress as a function of torque (T), radius (r), and polar moment of inertia (J).

Furthermore, if a torque is applied to a cylindrical specimen, the maximum torsional shearing stress (τ_{max}) can be found simply by measuring the radius (r) and calculating the polar moment of inertia (J) for a cylinder.

\[
\int \frac{2}{A} \rho \, d\rho = J, \quad \tau_{\text{max}} = \frac{T \cdot r}{J}
\]

16. \quad \tau_{\text{max}} = \frac{G \cdot r \cdot \phi}{L}

\rightarrow \phi = \frac{\tau_{\text{max}} \cdot L}{G \cdot r}

17. \quad \phi = \frac{T \cdot L}{G \cdot J}

\rightarrow G = \left(\frac{T}{\phi} \right) \cdot \left(\frac{L}{J} \right)

Hooke's Law previously defined the maximum torsional shearing stress (τ_{max}) as the product of the modulus of rigidity (G) and the maximum torsional shearing strain (γ_{max}), as seen in [11.]. In the equation derived in [07.], the maximum torsional shearing strain was defined as the product of the angle of deformation (φ) and the radius (r), divided by the length of the specimen (L). If the latter equation is plugged into the former in the place of the maximum torsional shearing strain (γ_{max}), an additional expression for the maximum torsional shearing stress (τ_{max}) is formed. The equation can be further manipulated, in order to give an expression for the angle of deformation (φ).

\[
\gamma_{\text{max}} = \frac{\phi \cdot r}{L}, \quad \tau_{\text{max}} = G \cdot \gamma_{\text{max}} \rightarrow \tau_{\text{max}} = \frac{G \cdot \phi \cdot r}{L} \rightarrow \tau_{\text{max}} = \frac{G \cdot r \cdot \phi}{L}
\]

When the expression which defines τ_{max} in equation [15.] is inserted in place of τ_{max} in the manipulated version of equation [16.], another expression for the angle of deformation (φ) is created. Furthermore, it defines the angle of deformation (φ) as a function of the torque (T), the length of the specimen (L), the modulus of rigidity (G), and the polar moment of inertia (J). The instances where the total radius (r) appeared in the equation cancel each other out, dropping out of the equation completely. Thus, when a torque (T) is applied to a given cylindrical specimen, the angle of deformation (φ) can be found by simply inserting measured and known values in for the length (L), the rigidity (G), and the polar moment of inertia (J). Moreover, the modulus of rigidity (G) can be found by manipulating the equation and substituting in the measured values. To go into additional specifics, G can be calculated by taking the slope of the plot of torque (T) vs. the angle of twist (φ) and multiplying that slope by the ratio of the length (L) to the polar moment of inertia (J).

\[
\tau_{\text{max}} = \frac{T \cdot r}{J}, \quad \phi = \frac{\tau_{\text{max}} \cdot L}{G \cdot r} \rightarrow \phi = \frac{T \cdot L}{G \cdot J}
\]
III. Tables:

<table>
<thead>
<tr>
<th>Table 1: Given/Theoretical Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of elasticity (psi)</td>
</tr>
<tr>
<td>3.00E+07</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2: 1018 Annealed Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specimen Dimensions</td>
</tr>
<tr>
<td>Diameter (inches)</td>
</tr>
<tr>
<td>0.371</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>G Measurement Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of twist (degrees)</td>
</tr>
<tr>
<td>16.71</td>
</tr>
<tr>
<td>18.43</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Yield Point Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of twist (degrees)</td>
</tr>
<tr>
<td>20.70</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ultimate Point Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of twist (degrees)</td>
</tr>
<tr>
<td>2021.06</td>
</tr>
</tbody>
</table>
### Table 3: 1018 Cold Worked Data

#### Specimen Dimensions

<table>
<thead>
<tr>
<th>Diameter (inches)</th>
<th>Length (inches)</th>
<th>Polar Moment of Inertia (in^4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.366</td>
<td>6.9</td>
<td>0.00176</td>
</tr>
</tbody>
</table>

#### G Measurement Values

<table>
<thead>
<tr>
<th>Angle of twist (degrees)</th>
<th>Angle of twist (radians)</th>
<th>Torque (in*lbf)</th>
<th>Modulus of rigidity (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.55</td>
<td>0.132</td>
<td>365.65</td>
<td>9.70E+06</td>
</tr>
<tr>
<td>8.88</td>
<td>0.155</td>
<td>423.09</td>
<td></td>
</tr>
</tbody>
</table>

#### Yield Point Values

<table>
<thead>
<tr>
<th>Angle of twist (degrees)</th>
<th>Angle of twist (radians)</th>
<th>Torque (in*lbf)</th>
<th>Shear Stress (psi)</th>
<th>Shear Strain (radians)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.63</td>
<td>0.203</td>
<td>532.53</td>
<td>55319</td>
<td>0.00538</td>
</tr>
</tbody>
</table>

#### Ultimate Point Values

<table>
<thead>
<tr>
<th>Angle of twist (degrees)</th>
<th>Angle of twist (radians)</th>
<th>Torque (in*lbf)</th>
<th>Shear Stress (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>257.03</td>
<td>4.486</td>
<td>957.07</td>
<td>99419</td>
</tr>
</tbody>
</table>

### Table 4: Comparison of G Values

<table>
<thead>
<tr>
<th>Annealed Percent Error</th>
<th>Cold Worked Percent Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>44.1%</td>
<td>15.9%</td>
</tr>
</tbody>
</table>
IV. Discussion of Results:

After being subjected to torsion, both the annealed and cold rolled rods eventually failed, as was expected. Furthermore, the annealed rod failed after deforming a significant amount more than the cold rolled rod. That is, the angle of rotation at which the annealed rod broke at was 35.274 radians, while the angle at which the cold rolled rod broke was 4.486 radians, as can be seen in Tables 2 and 3, respectively. The respective torques were 774.63 in*lbf and 957.07 in*lbf, showing that there was a much higher torque induced in the cold worked specimen. The ultimate stresses of the two had a similar relationship, at 77257 psi and 99419 psi, respectively, supporting the fact that the cold rolled rod was stronger. In addition, the modulus of rigidity for the annealed steel was substantially lower than the modulus for the cold rolled steel, at 6.45 * 10^6 psi, compared to 9.70 * 10^6 psi, respectively. These were calculated by finding the slopes of the linear parts of the curves, using the points in under G Measurement Values sub tables, and the dimensions, which was previously outlined in the theory. Putting this in perspective, it makes perfect sense, as, seen in the amount of rotation, it took longer for the annealed steel rod to break. This emphasizes the fact that the annealed specimen is far more ductile than the cold worked specimen, as previously theorized. The ultimate points of failure can be seen in Figures 2 and 4.

When analyzing the yield point, it is seen that the cold rolled specimen entered the plastic region after fewer rotations than the annealed steel did. This is supported by the higher brittleness of the cold rolled steel. The yield points for the annealed and cold rolled steel were significantly different; the cold rolled steel had a yield point which was difficult to distinguish, as can be seen in Figure 3. The yield point was thus estimated, using the principle of the offset method, wherein a line is envisioned parallel to the linear part of the curve, to the right of it by .2%. The yield point for the annealed steel was very easy to
pinpoint, as can be seen in Figure 1. The yield point for the annealed specimen occurred at a stress of 39331 psi and a strain of .00982 radians, and the yield point for the cold rolled specimen occurred at a stress of 55319 psi, and a strain .00538 radians, showing that their was a higher strain, at a lower stress, for the annealed, supporting the ductility and strengths of the different specimens once again.

An examination of the specimens after the torsion tests reveals certain details which support the mentioned theory. Due to a straight black line drawn on the rods, it is easily observed that the annealed specimen twisted more than the cold worked rod. Also, on the outside rods, a sort of powder was accumulated, due to impurities in the metal being released through the twisting. The rods were slightly hot after being twisted as well, due to the obvious fact that work was done on the rod. Furthermore, looking in on the ends of the rod, where it failed, fibers are seen to be curved around the axis of the rod. This correlates with the amount of twist, as the fibers in the annealed rod seemed to have a smaller radius of curvature.

When comparing the theoretical modulus of rigidity to calculated ones for each bar, there is appreciable error. The theoretical modulus of rigidity was found using the respective equation in the Calculations section, based off of modulus of elasticity and Poisson’s ratio, which are given theoretical values, as seen in Table 1. The declaration of error is true to a greater extent for the annealed steel rod, with a percent error of 44.1%, while the percent error for the cold rolled rod is but 15.9%, as shown in Table 4. If the definition of the modulus of rigidity is revisited, we see that this means that, the higher the modulus, the less likely the material is to deform before failure, and, in the context of the results, this can explain a higher modulus of rigidity of the material due to cold working. However, the annealed is still very low, meaning that the explanation for these errors is likely due to initial
error in how the experiment was performed. When the specimen was placed in the machine, an initial torque was applied. Attempts to reduce this torque caused the bar to loosen in the machine, causing failure of the experiment multiple times. This explains the unusual dip seen in Figure 1, and also may explain why the modulus of rigidity was so inaccurate. Since the specimen had already twisted, the properties of the bar may have changed somewhat, meaning the slope of the initial line would have been more closely accurate to the theoretical value. Due to taking the slope at rather high points on the linear part of the graph, there may have been some distortion in the modulus measurement. The cold rolled steel, on the other hand, having a modulus of rigidity closer to the theoretical value, did not fail a substantial amount, as did the annealed. However, there was still error; not all specimens can be exactly perfect, as factors during forming can alter it. Age and environment of the specimen can also affect this, showing why there is still an error in the modulus of rigidity. These factors could have played a role in the inaccuracy of the modulus of rigidity of the annealed specimen as well.
V. Experimental Data:

1018 Annealed Steel

Figure 1
Figure 2
1018 Cold Worked Steel

Figure 3
1018 Cold Worked Steel

Torque (lbf*in)

Rotation (deg)

Figure 4
VI. Calculations:

Polar Moment of Inertia:

\[ J = \frac{\pi d^4}{32} = \frac{\pi (0.371\text{in})^4}{32} = 0.00186\text{in}^4, \] where \( d \) is the diameter of the steel rod.

Theoretical Modulus of Rigidity:

\[ G = \frac{E}{2(1 + \nu)} = \frac{(30 \cdot 10^6 \text{ psi})^4}{2(1 + 0.3)} = 11.5 \cdot 10^6 \text{ psi}, \] where \( E \) is the theoretical modulus of elasticity, and \( \nu \) is Poisson's Ratio, which is a given, dimensionless ratio of lateral to axial strain.

Experimental Modulus of Rigidity:

\[ G = \frac{T \cdot L}{\phi \cdot J} = \frac{(270.92\text{in} \cdot \text{lbf} - 323.84\text{in} \cdot \text{lbf}) \cdot 6.825\text{in}}{(0.292\text{radians} - 0.322\text{radians}) \cdot 0.00186\text{in}^4} = 6.45 \cdot 10^6 \text{ psi}, \] where \( T \) is the expression for \( T/\phi \) is the slope of the linear portion of the plot, gathered from two measured points, \( L \) is the length of the specimen, and \( J \) is polar moment of inertia.

Yield Point Shear Stress:

\[ \tau_{max} = \frac{T \cdot d}{2 \cdot J} = \frac{394.35\text{in} \cdot \text{lbf} \cdot (0.371\text{in})}{2 \cdot (0.00186\text{in}^4)} = 30331 \text{ psi}, \] where \( T \) is the measured torque at the yield point, \( d \) is the diameter of the steel rod, and \( J \) is the polar moment of inertia.

Yield Point Shear Strain:
\[
\gamma = \frac{\phi \cdot d}{2 \cdot L} = \frac{(0.36 \text{ radians}) \cdot (0.37 \text{ in})}{2 \cdot (6.825 \text{ in})} = 0.00982 \text{ radians},
\]

where \( \phi \) is the measured angle of twist at the yield point, \( d \) is the diameter of the steel rod, and \( L \) is the length of the steel rod.

**Ultimate Shear Stress:**

\[
\tau_{\text{max}} = \frac{T \cdot d}{2 \cdot J} = \frac{(774.63 \text{ in} \cdot \text{lb}) \cdot (0.37 \text{ in})}{2 \cdot (0.00186 \text{ in}^4)} = 77257 \text{ psi},
\]

where \( T \) is the measured torque at the ultimate point, \( d \) is the diameter of the steel rod, and \( J \) is the polar moment of inertia.

**Angle of Twist (radians):**

\[
\phi (\text{radians}) = \phi (\text{deg}) \cdot \frac{\pi \cdot \text{radians}}{180 \cdot \text{deg}} = 20.70 \text{ deg} \cdot \frac{\pi \cdot \text{radians}}{180 \cdot \text{deg}} = 0.36 \text{ radians},
\]

where \( \phi (\text{radians}) \) is the angle of twist, measured in radians, and \( \phi (\text{deg}) \) is the angle of twist, measured in degrees.